IMPORTANT QUESTIONS FOR SECTION B:

1- Find the solution set of the following equations

$|2a-3|=7$

Solution:

$|2a-3|=7$

$2a-3=\pm 7$

Either

$2a-3=7$

$2a=7+3$

$2a=10$

$a=\frac{10}{2}$

$a=5$

Or

$2a-3=-7$

$2a=-7+3$

$2a=-4$

$a=-\frac{4}{2}$

$a=-2$

The Solution Set $=\{-2,5\}$

2- Marks obtained by the students in chemistry are given. Find: Arithmetic Mean

Marks Obtained	25-29	30-34	35-39	40-44	45-49
No. of Students	9	18	35	17	5

Solution:

Class Intervals	**Mid-Points** $(\boldsymbol{x})$	**Frequency** $(\boldsymbol{f})$	$\boldsymbol{fx}$
$25-29$	27	9	$9\times 27=243$
$30-34$	32	18	$18\times 32=576$
$35-39$	37	35	$35\times 37=1295$
$40-44$	42	17	$17\times 42=714$
$45-49$	47	5	$5\times 47=235$
$Total$		$\sum f=84$	$\sum fx=3063$

$$\bar{x}=\frac{\sum fx}{\sum f}$$

$$=\frac{3063}{84}$$

$$=36.46$$

3- Find the solution set of the following equations

$\sqrt{12x-4}=\sqrt{4x+8}$

Solution:

$\sqrt{12x-4}=\sqrt{4x+8}$

Taking square on both sides

$\left(\sqrt{12x-4}\right)^2=\left(\sqrt{4x+8}\right)^2$

$12x-4=4x+8$

$12x - 4x = 8 + 4$

$8x = 12$

$x = \frac{12}{8}$

$x = \frac{6}{4}$

$x = \frac{3}{2}$

The Solution set $= \left\{\frac{3}{2}\right\}$

4- Find the solution set of the following equations by using Quadratic formula :

$2b^2 - 7b + 5 = 0$

Solution:

$2b^2 - 7b + 5 = 0$

Here $a = 2, b = -7$ and $c = 5$

By using quadratic formula :

$b = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$b = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(5)}}{2(2)}$

$b = \frac{7 \pm \sqrt{49 - 40}}{4}$

$b = \frac{7 \pm \sqrt{9}}{4}$

$b = \frac{7 \pm 3}{4}$

Either

$b = \frac{7+3}{4}$

$b = \frac{10}{4}$

$b = \frac{5}{2}$

Or

$b = \frac{7-3}{4}$

$b = \frac{4}{4}$

$b = 1$

The Solution set $= \left\{1, \frac{5}{2}\right\}$

5- Eliminate t from the following equations by substitution method.

$$\boldsymbol{v_f = v_i + gt, \quad s = v_i t + \frac{1}{2} g t^2}$$

Solution:

The given equations are

$$v_f = v_i + gt \quad \ldots\ldots\ldots\ldots\ldots(i)$$

$$s = v_i t + \frac{1}{2} g t^2 \quad \ldots\ldots\ldots\ (ii)$$

From equation (i)

$$v_f = v_i + gt$$

$$v_f - v_i = gt$$

$$\frac{v_f - v_i}{g} = t$$

$$t = \frac{v_f - v_i}{g} \ldots\ldots\ldots\ldots\ldots(iii)$$

Put the value of t in equ(ii)

$Equ(ii) \Rightarrow$

$$s = v_i\left(\frac{v_f - v_i}{g}\right) + \frac{1}{2} g \left(\frac{v_f - v_i}{g}\right)^2$$

$$s = \frac{v_i(v_f - v_i)}{g} + \frac{g}{2}\frac{(v_f - v_i)^2}{g^2}$$

$$s = \frac{v_i v_f - v_i^2}{g} + \frac{(v_f - v_i)^2}{2g}$$

$$s = \frac{2(v_i v_f - v_i^2) + (v_f - v_i)^2}{2g}$$

$$s = \frac{2v_i v_f - 2v_i^2 + v_f^2 - 2v_i v_f + v_i^2}{2g}$$

$$s = \frac{v_f^2 - v_i^2}{2g}$$

$$\boxed{2gs = v_f^2 - v_i^2}$$

The value of "t" has eliminated.

6- Find relations independent of x from the following equations by the use of formulae.

$$x + \frac{1}{x} = 2p\,, \quad x - \frac{1}{x} = 2q + 1$$

Solution:

The given equations are

$$x + \frac{1}{x} = 2p ----(i)$$

$$x - \frac{1}{x} = 2q + 1 ----(ii)$$

Taking square on both sides on equation (i)

$$\left(x + \frac{1}{x}\right)^2 = (2p)^2$$

$$\because (a+b)^2 = a^2 + 2ab + b^2$$

$$x^2 + 2(x)\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2 = 4p^2$$

$$x^2 + 2 + \frac{1}{x^2} = 4p^2$$

$$x^2 + \frac{1}{x^2} = 4p^2 - 2 ----(iii)$$

Taking square on both sides on equation (ii)

$$\left(x - \frac{1}{x}\right)^2 = (2q+1)^2$$

$$\because (a-b)^2 = a^2 - 2ab + b^2$$

$$x^2 - 2(x)\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2 = (2q)^2 + 2(2q)(1) + (1)^2$$

$$x^2 - 2 + \frac{1}{x^2} = 4q^2 + 4q + 1$$

$$x^2 + \frac{1}{x^2} = 4q^2 + 4q + 1 + 2$$

$$x^2 + \frac{1}{x^2} = 4q^2 + 4q + 3 ----(iv)$$

By comparing equ(i) and equ(ii) , we get :

$$4p^2 - 2 = 4q^2 + 4q + 3$$

$$4p^2 - 2 - 4q^2 - 4q - 3 = 0$$

$$4p^2 - 4q^2 - 4q - 5 = 0$$

$$\boxed{4p^2 - 4q^2 - 4q - 5 = 0}$$

The value of "x" has eliminated.

7- Eliminate *t* from the following with the help of formulae:

$$\frac{x}{p} = \frac{1+t^2}{2t} \quad , \quad \frac{y}{q} = \frac{1-t^2}{2t}$$

Solution: The given equations are

$$\frac{x}{p} = \frac{1+t^2}{2t} ----(i)$$

$$\frac{y}{p} = \frac{1-t^2}{2t} ----(ii)$$

From equation (i)

$$\frac{x}{p} = \frac{1+t^2}{2t}$$

Squaring on both sides

$$\left(\frac{x}{p}\right)^2 = \left(\frac{1+t^2}{2t}\right)^2$$

$$\because (a+b)^2 = a^2 + 2ab + b^2$$

$$\frac{x^2}{p^2} = \frac{(1)^2 + 2(1)(t^2) + (t^2)^2}{4t^2}$$

$$\frac{x^2}{p^2} = \frac{1 + 2t^2 + t^4}{4t^2} \dots\dots\dots\dots\dots (iii)$$

From equation (ii)

$$\frac{y}{p} = \frac{1-t^2}{2t}$$

Squaring on both sides

$$\left(\frac{y}{p}\right)^2 = \left(\frac{1-t^2}{2t}\right)^2$$

$$\because (a-b)^2 = a^2 - 2ab + b^2$$

$$\frac{y^2}{q^2} = \frac{(1)^2 - 2(1)(t^2) + (t^2)^2}{4t^2}$$

$$\frac{y^2}{q^2} = \frac{1 - 2t^2 + t^4}{4t^2} \dots\dots\dots\dots\dots (iv)$$

Subtracting equation (iv) from equation (iii), we get

$$\frac{x^2}{p^2} - \frac{y^2}{q^2} = \frac{1 + 2t^2 + t^4}{4t^2} - \frac{1 - 2t^2 + t^4}{4t^2}$$

$$\frac{x^2}{p^2}-\frac{y^2}{q^2}=\frac{(1+2t^2+t^4)-(1-2t^2+t^4)}{4t^2}$$
$$\frac{x^2}{p^2}-\frac{y^2}{q^2}=\frac{1+2t^2+t^4-1+2t^2-t^4}{4t^2}$$
$$\frac{x^2}{p^2}-\frac{y^2}{q^2}=\frac{4t^2}{4t^2}$$
$$\boxed{\frac{x^2}{p^2}-\frac{y^2}{q^2}=1}$$

The value of "t" has eliminated.

8- If $\frac{a}{b+c}=\frac{b}{c+a}=\frac{c}{a+b}$ and a + b + c ≠ 0 prove that $a=b=c$.

Proof: From the theorems on proportion

$$\text{Each fraction} = \frac{a+b+c}{b+c+c+a+a+b}$$
$$=\frac{a+b+c}{2a+2b+2c}$$
$$=\frac{(a+b+c)}{2(a+b+c)}$$
$$=\frac{1}{2}$$

Now,

$$\text{each fraction} = \frac{1}{2}$$

So, $$\frac{a}{b+c}=\frac{1}{2}$$
$$2a=b+c$$

Adding 'a' on both sides

$$2a+a=a+b+c$$
$$3a=a+b+c$$
$$a=\frac{a+b+c}{3} \longrightarrow (i)$$
$$\frac{b}{c+a}=\frac{1}{2}$$
$$2b=c+a$$

Adding 'b' on both sides

$$2b+b=a+b+c$$
$$3b=a+b+c$$

$$b = \frac{a+b+c}{3} \longrightarrow (ii)$$

$$\frac{c}{a+b} = \frac{1}{2}$$

$$2c = a+b$$

Adding 'c' on both sides

$$2c + c = a+b+c$$

$$3c = a+b+c$$

$$c = \frac{a+b+c}{3} \longrightarrow (iii)$$

Comparing equation $(i), (ii) and (iii)$, we get

$$\boxed{a = b = c}$$ Proved

9- If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ prove that $\frac{a^4b^2+a^2e^2-e^4f}{b^6+b^2f^2-f^5} = \frac{a^4}{b^4}$

Proof:

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

Then $\frac{a}{b} = k \Rightarrow a = bk$

$$\frac{c}{d} = k \Rightarrow c = dk$$

$$\frac{e}{f} = k \Rightarrow e = fk$$

$$\text{L.H.S} = \frac{a^4b^2 + a^2e^2 - e^4f}{b^6 + b^2f^2 - f^5}$$

$$= \frac{(bk)^4b^2 + (bk)^2(fk)^2 - (fk)^4f}{b^6 + b^2f^2 - f^5}$$

$$= \frac{b^6k^4 + b^2f^2k^4 - f^5k^4}{b^6 + b^2f^2 - f^5}$$

$$= \frac{k^4(b^6 + b^2f^2 - f^5)}{b^6 + b^2f^2 - f^5}$$

$$= k^4$$

$$\text{R.H.S} = \frac{a^4}{b^4}$$

$$= \frac{(bk)^4}{b^4}$$

$$= \frac{b^4k^4}{b^4}$$

$\text{R.H.S} = k^4$

$\text{R.H.S} = \text{L.H.S}$

Hence

$$= \frac{a^4b^2 + a^2e^2 - e^4f}{b^6 + b^2f^2 - f^5} = \frac{a^4}{b^4}$$

10- Solve the following for x:

$\frac{(x+3)^2+(x-1)^2}{(x+3)^2-(x-1)^2} = \frac{5}{4}$

Solution: $\frac{(x+3)^2+(x-1)^2}{(x+3)^2-(x-1)^2} = \frac{5}{4}$

By componendo and dividendo property

$$\frac{\{(x+3)^2+(x-1)^2\}+\{(x+3)^2-(x-1)^2\}}{\{(x+3)^2+(x-1)^2\}-\{(x+3)^2-(x-1)^2\}} = \frac{5+4}{5-4}$$

$$\frac{(x+3)^2+(x-1)^2+(x+3)^2-(x-1)^2}{(x+3)^2+(x-1)^2-(x+3)^2+(x-1)^2} = \frac{9}{1}$$

$$\frac{2(x+3)^2}{2(x-1)^2} = 9$$

$$\frac{(x+3)^2}{(x-1)^2} = 9$$

$$\left(\frac{x+3}{x-1}\right)^2 = 9$$

Taking square root on both sides

$$\sqrt{\left(\frac{x+3}{x-1}\right)^2} = \sqrt{9}$$

$$\frac{x+3}{x-1} = 3$$

$$x+3 = 3(x-1)$$

$$x+3 = 3x-3$$

$$3+3 = 3x-x$$

$$6 = 2x$$

$$x = \frac{6}{2}$$

$\boxed{x = 3}$ Ans.

12- Prove that Congruent Chords of a circle are equidistant from the centre.

Given:

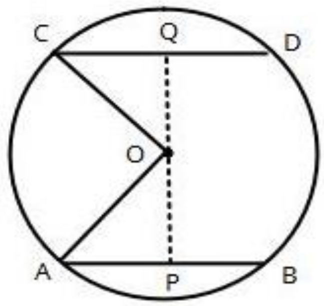
$\overline{AB}$ and $\overline{CD}$ are two congruent chords in a circle with centre O. $\overline{OP} \perp \overline{AB}$ and $\overline{OQ} \perp \overline{CD}$

To Prove that:

$\overline{OP} \cong \overline{OQ}$

Construction:

Draw $\overline{OA}$ and $\overline{OC}$



Proof:

Statements	Reasons
1. $m\overline{AP} = \frac{1}{2} m\overline{AB}$	1. $\overline{OP} \perp \overline{AB}$ (Theorem)
2. $m\overline{CQ} = \frac{1}{2} m\overline{CD}$	2. $\overline{OQ} \perp \overline{CD}$ (Theorem)
3. But $m\overline{AB} = m\overline{CD}$	3. Given
4. $\therefore m\overline{AP} = m\overline{CQ}$	4. Transitive property of equation.
5. In $\Delta AOP \leftrightarrow \Delta COQ$	5.
i. $\overline{AO} \cong \overline{CO}$	i. Radial segments of same circle
ii. $\overline{AP} \cong \overline{CQ}$	ii. Proved above
iii. $\angle APO \cong \angle CQO$	iii. Right angles
6. $\Delta AOP \cong \Delta COQ$	6. In right $\Delta s, H.S \cong H.S$
7. $\overline{OP} \cong \overline{OQ}$	7. By the congruence of Δs
	Q.E.D

13- **Construct the following triangles**

$\Delta DEF \text{ such that } m\overline{DE} = 8.7\ cm\ , m\overline{EF} = 4.3cm \text{ and } m\angle E = 45°.$

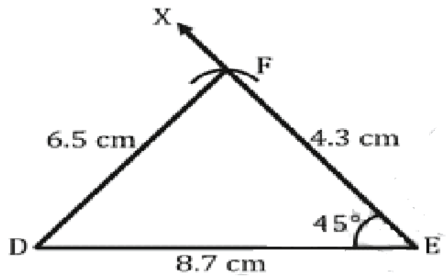
Steps of construction :

(i) Draw the longest line segment $\overline{DE}$of measure 8.7 cm.

(ii) On point E draw an angle XEF of measure 45°.

(iii) Cut $\overline{EF}$ of measure 4.3 cm from $\overline{EX}$.

(iv) Join $\overline{DF}$so we get the required triangle DEF.



14- Prove that: $\frac{cot^2\alpha - 1}{1 + cot^2\alpha} = 2\ cos^2\ \alpha - 1$

Solution:

Taking L.H.S

$$= \frac{cot^2\alpha - 1}{1 + cot^2\alpha}$$

$$\because \frac{\cos\alpha}{\sin\alpha} = \cot\alpha$$

$$= \frac{\frac{\cos^2\alpha}{\sin^2\alpha} - 1}{1 + \frac{\cos^2\alpha}{\sin^2\alpha}}$$

$$= \frac{\frac{\cos^2\alpha - \sin^2\alpha}{\sin^2\alpha}}{\frac{\sin^2\alpha + \cos^2\alpha}{\sin^2\alpha}}$$

$$= \frac{\cos^2\alpha - \sin^2\alpha}{\sin^2\alpha + \cos^2\alpha}$$

$$\because \sin^2\alpha + \cos^2\alpha$$

$= \frac{\cos^2 \alpha - \sin^2 \alpha}{1}$

$= \cos^2 \alpha - \sin^2 \alpha$

$\because \mathbf{\sin^2 \alpha + \cos^2 \alpha = 1}$

$\because \mathbf{\sin^2 \alpha = 1 - \cos^2 \alpha}$

$= \cos^2 \alpha - (1 - \cos^2 \alpha)$

$= \cos^2 \alpha - 1 + \cos^2 \alpha$

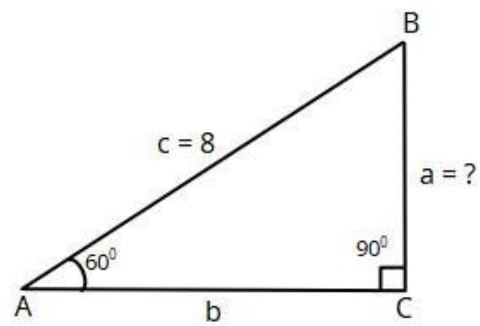
$= \cos^2 \alpha + \cos^2 \alpha - 1$

$= 2\cos^2 \alpha - 1$

= R.H.S (Proved)

15- A ladder 8 meters long reaches a point on a wall such that it makes angle of 60^0 with the ground. Find the height of the point on the wall where the top end of the ladder touches it.

Solution:
Consider a right angle ΔACB



Where,

height of wall , $a = ?$

Using,

$\sin\theta = \frac{Perpendicular}{Hypotenuse}$

$\sin\theta = \frac{a}{c}$

$\sin 60^0 = \frac{a}{8}$

As we know that,

$\sin 60^0 = \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{2} = \frac{a}{8}$

By cross multiplication

$8 \times \sqrt{3} = a \times 2$

$8\sqrt{3} = 2a$

$a = \frac{8\sqrt{3}}{2}$

$a = 4\sqrt{3}$

$\therefore \sqrt{3} = 1.7320$

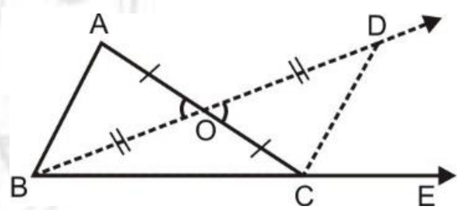
$a = 4(1.7320)$

$\boxed{a = 6.928\, m}$

16- If a side of a triangle is extended, the exterior angle so formed is, in measure, greater than either of the two interiors opposite angles.

Given

ΔABC with exterior angle ACE.



To Prove

$m\angle ACE > m\angle A$ and $m\angle ACE > m\angle B$.

Construction

Let O be the mid-point of $\overline{AC}$. Draw $\overrightarrow{BO}$ and cut off $\overline{OD} \cong \overline{BO}$, and draw $\overline{CD}$.

Proof

#	Statements	Reasons
1.	In $\Delta AOB \leftrightarrow \Delta COD$ (i) $\overline{AO} \cong \overline{CO}$ (ii) $\angle AOB \cong \angle COD$ (iii) $\overline{BO} \cong \overline{DO}$	 (i) Construction. (ii) Vertical angles. (iii) Construction.
2.	So, $\Delta AOB \cong \Delta COD$	S.A.S. Postulate.
3.	$\therefore m\angle A \cong m\angle OCD$	By the congruence of triangles.
4.	But $m\angle ACE = m\angle OCD + m\angle DCE$	By angle addition postulate.
5.	So, $m\angle ACE > m\angle OCD$	Whole is greater than its part.
6.	$\therefore m\angle ACE > m\angle A$	By (3).
7.	Similarly, $m\angle ACE > m\angle B$	By the above process.

Q. E. D.

17- If a transversal intersects two coplanar lines such that the pair of alternate angles is congruent, then the lines are parallel.

Given

$\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are two coplanar lines and transversal $\overleftrightarrow{EF}$ cuts them at point P and Q respectively, such that $\angle 1 \cong \angle 2$ (pair of alternate angles).

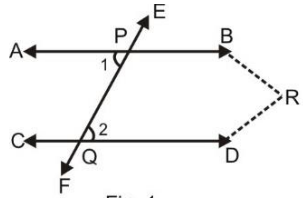


Fig. 1

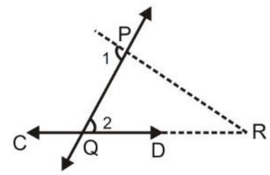


Fig. 2

To Prove

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

Proof

If $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are not parallel then being coplanar, they will meet at a point (say at R), and PQR will be a triangle and look like Fig. 2.

#	Statements	Reasons
1.	In ΔPQR, $\angle 1$ is exterior angle and $\angle 2$ is interior angle opposite to $\angle 1$.	By definition.
2.	So, $m\angle 1 > m\angle 2$	By Theorem 2.
3.	But $m\angle 1 = m\angle 2$	Given.
4.	Statements (2) and (3) cannot be true simultaneously.	Trichotomy Property of order relation.
5.	Hence $m\angle 1 = m\angle 2$ and $\overline{AB}$ and $\overline{CD}$ do not intersect each other.	Contradiction in assumption.
6.	Or $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	Because they are coplanar and do not intersect.

Q. E. D.